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Moment estimation in Auerbach-Kotlikoff models:

How well do they match the data?

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Moment Estimation in Auerbach-Kotlikoff Models - How Well Do They Match the Data?*

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Preliminary. Comments Welcome

Abstract

Despite their widespread use for the analysis of economic questions, a formal and systematic calibration methodology has not yet been developed for Auerbach-Kotlikoff (Auerbach and Kotlikoff 1987) overlapping generations (AK-OLG) models. Calibration as estimation in macroeconomics involves choosing free parameters by matching moments of simulated models with those of the data. This paper maps this approach into the framework of AK-OLG models. The paper further evaluates the back-fitting properties of three different versions of a prototype AK-OLG model along a number of dimensions of mostly US data for the time period 1960-2003.

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1 Introduction

Since almost a quarter of a century, Auerbach-Kotlikoff type overlapping generations (AK-OLG) models (Auerbach, Kotlikoff, and Skinner 1983; Auerbach and Kotlikoff 1987) have been applied to the analysis of economic questions. Kotlikoff (1998) provides a review of the (earlier) literature and summarizes avenues of future research. Among the more recent developments in the AK-OLG literature are the inclusion of realistic demographic profiles and the extension towards multi-country versions of these models (Bommier and Lee 2003; INGENUE 2001; Börsch-Supan, Ludwig, and Winter 2004; Fehr, Jokisch, and Kotlikoff 2004).[1] Such extensions have moved AK-OLG models from being mere analytical models applied to public finance questions into the direction of forecasting tools.

These recent developments necessitate a careful evaluation of AK-OLG models with regard to their fit to long time series of macroeconomic data. This in turn requires a formal procedure to determine values of structural model parameters, which is referred to as calibration.[2] The purpose of the present paper is twofold: First, a new, systematic calibration procedure is developed for large-scale AK-OLG models in outside steady state situations. The suggested approach is to estimate structural model parameters by a formal matching of moments procedure. Second, the fit of a prototype AK-OLG model to long time series of macroeconomic data is evaluated and the relative performance of different model features is compared. Model evaluation relates to the alternative interpretation of calibration that has been used in the literature as a way of *testing* an economic model.[3] To the best of my knowledge, this paper is the first of its kind to provide such detailed analyses of both these aspects for AK-OLG models.

Standard calibration procedures of AK-OLG models stratify the set of all structural model parameters into two sets, *predetermined* and *free* parameters. *Predetermined* parameter values are set by reference to (estimates of) other studies. Values of *free* parameters are determined by informally matching moments.

The use of *predetermined* parameters has been criticized with the notion that statistical inference depends on the structure of the econometric model. Parameter values are therefore not easily transferable from one particular model to another (Hansen and Heckman 1996). Furthermore, and as emphasized by Gregory and Smith (1990), estimation of the subset of free parameters depends on

[1]It is beyond the scope of this paper to provide a similar review on the more recent literature as in Kotlikoff (1998). Among other model features that have recently been added are, e.g., within generation heterogeneity and idiosyncratic as well as aggregate uncertainty (Imrohoroglu, Imrohoroglu, and Joines 1995; Conesa and Krueger 1999; Altig et al. 2001; Krueger and Kubler 2003).

[2]Kim and Pagan (1995) provide a review of the literature on "calibration as estimation" (Gregory and Smith 1990).

[3]This alternative interpretation of calibration is more in line with the interpretation of calibration by Kydland and Prescott (1982). Canova and Ortega (1999) provide a review of the literature on "calibration as testing" (Gregory and Smith 1991).

the values of predetermined parameters. While not desirable, it is often unavoidable to rely on predetermined parameters. Here, the selection of predetermined parameters is regarded as exogenous but the sensitivity of the effects of errors in it can be shown to be low.[4]

The standard procedure of informally matching moments to determine values of *free* parameters used in the AK-OLG literature is a mix of the following two approaches. The first is to focus exclusively on observations of a base year.[5] Obviously the procedure has the drawback that observations in any time period are just realizations of an (unknown) stochastic data generating process and (or) are measured with error. The second approach calibrates the model such as to (informally) match long term averages of statistical data. While this second procedure to a large extent overcomes the deficiencies of the first, growth rates of variables are usually regarded as predetermined. Being informal, both approaches do further not take account of the sampling uncertainty of structural model parameters.

One reason for the lack of more sophisticated econometric techniques in AK-OLG calibration is certainly conceptually grounded in the deterministic nature of these models. Accordingly, observations of a base year suffice to determine values of structural model parameters. This paper deviates from this view by augmenting deterministic dynamic AK-OLG models with additional random components as in the early work on CGE models by Jorgenson (1984) and Mansur and Whalley (1984). *Free* structural model parameters are estimated using a method of moments methodology that sets to zero the average discrepancy (discrepancy function) between actual and predicted (simulated) values along pre-specified dimensions. This is by no means a trivial task since a number of the moment conditions do not have closed form solutions and the estimation method therefore has to rely on numerical simulation.[6] Adopting the terminology of Gregory and Smith (1990), the suggested calibration procedure can therefore be understood as a restricted method of simulated moments procedure, where the restrictions stem from the choice of predetermined parameters.

Model evaluation is by means of two approaches. First, graphical inspection is used to study the discrepancies between the time paths of actual and simulated data. While this way of testing the model provides most information, it has been criticized in the literature as being too *informal* (Hansen and Heckman 1996) since a formal metric to evaluate the distance between actual and simulated data is not provided. In order to provide such a formal metric, this paper adopts the framework of Christiano and Eichenbaum (1992) who map estimation and

[4]Results on such a sensitivity analysis are available from the author upon request.

[5]Abdelkhalek and Dufour (1998), within the context of a different type of CGE model, justify this procedure by noting that long time series on economic variables are often not available, e.g., for developing countries.

[6]The implied costs of the estimation procedure may be another reason for the lack of more sophisticated econometric calibration techniques in the AK-OLG literature.

testing of a Real Business Cycle (RBC) model into a modification of Hansen's (1982) GMM framework in an elegant way. While more emphasis will be put on model evaluation by graphical means, the formal criteria are regarded as a useful complement of the graphical analysis that validate its findings from a statistical perspective.

The remainder of the paper is structured as follows. Section 2 presents the calibration methodology. Section 3 contains a description of the key features of a prototype AK-OLG model used as an illustration of the calibration procedure. Section 4 presents the results. Finally, Section 5 draws conclusions from these findings.

2 The Calibration Procedure

Computable general equilibrium models such as AK-OLG models can be represented by the following function F

$$y_t = F(Y, X, \Psi^c) \quad \forall t = T_1, ..., T_2. \tag{1}$$

$X = \{\{x_{t,i}\}_{i=1}^{m}\}_{t=T_1}^{T_2}$ is a collection of exogenous and $Y = \{\{y_{t,i}\}_{i=1}^{n}\}_{t=T_1}^{T_2}$ is a collection of endogenous variables. This general representation allows lagged and future endogenous (exogenous) variables to enter the model. They are determined (given) for a simulation period of length $T_1 + T_2 + 1$ starting from the initial date $T_1 < 1$ and ending at the final date $T_2 \geq T$, whereas data are only observed for the period $1, ..., T$.

$\Psi^c \in \Gamma \subset \mathbb{R}^c$ denotes the $c \times 1$ vector of structural model parameters which are referred to as calibration parameters. Define by Ψ^p the vector of p *predetermined* parameters and by Ψ^e the vector of e *estimated* parameters, where $\Psi = [(\Psi^p)', (\Psi^e)']'$. While Ψ^p and Ψ^e are not fundamentally different from a theoretical viewpoint, they are treated differently in standard calibration of CGE models. Predetermined parameters, Ψ^p, are set by reference to other studies and are usually elasticity parameters that describe behavioral functions, whereas estimated parameters, Ψ^e, are usually scale or share parameters (Abdelkhalek and Dufour 1998). Note that, in the extreme cases, either of the two vectors may be empty. Hence, if $p = 0$ all parameters are determined by estimation and if $e = 0$ all parameters are predetermined.

To simplify notation, the above equation can be rewritten as

$$y_t = f(y_t, x_t, \Psi) = h_t(\Psi^e) \quad \forall t = T_1, ..., T_2 \tag{2}$$

such that only contemporaneous variables enter the right-hand side of the equation.

2.1 A Modified GMM Framework

Structural model parameters, Ψ^e, are estimated by unconditional matching of moments as in Christiano and Eichenbaum (1992). e moment conditions will be used to estimate the elements of Ψ^e (exactly identified case of GMM estimation). In anticipation of further results, it is however useful to start with the more general case of GMM estimation where the total number of moment conditions, r, exceeds the number of parameters, e.

Let

$$u_t^e(\Psi^e) = y_t^e - h_t^e(\Psi^e) \tag{3}$$

be an $e \times 1$ vector. Assume that q additional moment conditions are given and define by

$$u_t^q(\Psi^e) = y_t^q - h_t^q(\Psi^e) \tag{4}$$

a $q \times 1$ vector, where $r = e + q$. Further define the overall GMM error as $u_t = [(u_t^e)', (u_t^q)']'$.

Under the assumption that the model is correctly specified, the restrictions on the GMM error can be written as

$$\mathbb{E}[u_t(\Psi^{e,0})] = 0, \tag{5}$$

where $\Psi^{e,0}$ denotes the vector of true values.

Denote the sample averages of u_t as

$$g_T(\Psi^e) \equiv \frac{1}{T}\sum_{t=1}^{T} u_t(\Psi^e), \qquad g_T(\Psi^e) = [g_T^e(\Psi^e)', g_T^q(\Psi^e)']', \tag{6}$$

where $T < T_2$ is the sample size. Hansen's 1982 GMM estimator $\widehat{\Psi}_T^e$ is then defined as

$$\widehat{\Psi}_T^e = \arg\min_{\Psi^e} g_T(\Psi^e) W g_T(\Psi^e) \tag{7}$$

for some weighting matrix W.

Calibration as unconditional moment estimation of Ψ^e and testing of the model by informal methods can be understood as restricted GMM estimation with the restriction on W given by

$$W = \begin{bmatrix} I_{e\times e} & 0_{e\times q} \\ 0_{q\times e} & 0_{q\times q} \end{bmatrix}, \tag{8}$$

compare, e.g., Marcet (1994). In other words, while e moment conditions are used to estimate e structural model parameters, the remaining q moment conditions are used to test the model. By the above restriction on W, tests of the model based on g_T^q are necessarily informal.

Following Christiano and Eichenbaum (1992) a formal framework for testing the model - without leaving the "philosophy" of calibration of exactly matching

e moments to estimate Ψ^e - is developed as follows. Define a $q \times 1$ vector of additional model parameters, Ψ^q, and by $\Psi = [(\Psi^e)', (\Psi^q)']'$ the $r \times 1$ vector collecting all parameters. Further, rewrite the GMM errors in equation 4 as

$$u_t^q(\Psi) = \underbrace{y_t^q - \Psi^q}_{u_{t,1}^q(\Psi^q)} - \underbrace{(h_t^q(\Psi^e) - \Psi^q)}_{u_{t,2}^q(\Psi)}, \quad \forall t = 1, ..., T \quad . \tag{9}$$

and define the sample averages of the GMM errors $u_{t,1}^q(\Psi^q)$ and $u_{t,2}^q(\Psi)$ as

$$g_{T,1}^q(\Psi^q) \equiv \frac{1}{T}\sum_{t=1}^{T} u_{t,1}^q(\Psi^q) \qquad g_{T,2}^q(\Psi) \equiv \frac{1}{T}\sum_{t=1}^{T} u_{t,2}^q(\Psi). \tag{10}$$

Notice that $g_{T,1}^q(\Psi^q)$ measures the average discrepancy between actual variables, y_t^q, from the parameters Ψ^q - i.e., Ψ^q are the sample averages of y_t^q -, whereas $g_{T,2}^q(\Psi)$ measures the average discrepancy between simulated variables, $h_t^q(\Psi^e)$, and the parameters Ψ^q.

The GMM estimator of the $r \times 1$ vector $\widehat{\Psi}$ is now derived from the $r \times 1$ moment conditions $g_T(\Psi) = [(g_T^e(\Psi^e))', (g_{T,1}^q(\Psi^q))']'$ and defined by

$$g_T(\widehat{\Psi}_T) = 0, \tag{11}$$

i.e., the weighting matrix corresponding to the representation in equation 7 is an identity matrix, $W = I_{r \times r}$. The role $g_{T,2}^q(\Psi)$, will be addressed below.

Assume, as in the seminal contribution by Hansen (1982), that u_t, are strictly stationary for all possible Ψ. Then $\widehat{\Psi}_T$ is asymptotically normally distributed,

$$\sqrt{T}(\widehat{\Psi}_T - \Psi^0) \sim N(0, V), \tag{12}$$

where

$$V = D^{-1}S(D')^{-1} \tag{13}$$

and

$$D = \mathbb{E}\left[\frac{\partial g_T(\Psi)}{\partial \Psi'} \mid_{\Psi=\Psi^0}\right] = 0. \tag{14}$$

S is the positive semi-definite spectral density at frequency 0 of $u_t(\Psi^0)$ defined by

$$S = \sum_{l=-\infty}^{\infty} C_l \;\text{ where }\; C_l = \mathbb{E}[u_t(\Psi)u_{t-l}(\Psi)']. \tag{15}$$

Inference is based on replacing D and S with estimators, hence

$$\widehat{V}_T = \widehat{D}_T^{-1}\widehat{S}_T(\widehat{D}_T')^{-1} \tag{16}$$

and $\widehat{\Psi}$ can be treated approximately as

$$\widehat{\Psi}_T \sim N\left(\Psi^0, \text{var}(\widehat{\Psi})\right), \qquad \text{var}(\widehat{\Psi}) = \widehat{V}/T. \tag{17}$$

Considering formal tests of the model, define by $f^s(\Psi^0)$ a function that maps $\mathbb{R}^r$ into the $s \times 1$ vector 0_s. Then $f^s(\Psi^0) = 0_s$ presents s hypothesis each of which potentially involves all elements of Ψ^0. As shown in Christiano and Eichenbaum (1992), the statistic

$$J = f(\widehat{\Psi})' \text{var} f(\widehat{\Psi})^{-1} f(\widehat{\Psi}), \tag{18}$$

where

$$\text{var} f(\widehat{\Psi}) = f'(\widehat{\Psi}) \text{var}(\widehat{\Psi}) f'(\widehat{\Psi})' \tag{19}$$

is asymptotically χ^2-distributed with s degrees of freedom, also see Eichenbaum, Hansen, and Singleton (1988); Christiano and Den Haan (1996). For example, tests involving all the additional q parameters can be mapped into this framework if $f(\widehat{\Psi}) = g_{T,1}(\widehat{\Psi})$, hence $s = q$. Equation 18 takes into account the joint sampling uncertainty of the model parameter estimates and the moments of the data and represents a formal theory of inference that may serve as a useful complement of the informal and mostly graphical model evaluation procedure.

2.2 The Case of Non-Stationarity

The assumption of strict stationarity of u_t is restrictive since economic models often evolve variables that are trending over time as is also the case for the economic model described in Section 3. Cases with trending variables have been considered by Eichenbaum and Hansen (1990) and Ogaki (1993, 1999). An obvious solution to the non-stationarity is to transform variables of the economic model such that the transformed variables used in the econometric application are stationary as in the study by Hansen and Singleton (1982). However, it may not always be feasible to rewrite an economic model as such.

An alternative has been discussed by Eichenbaum and Hansen (1990) and by Ogaki (1993). Eichenbaum and Hansen consider two types of trends, a deterministic and a stochastic trend. For the economic application in this paper, the deterministic trend specification is of relevance. Suppose that a variable Z_t satisfies

$$Z_t = Z_0 \exp(\gamma^z t + u_t^z),$$

and hence that

$$z_t = \ln Z_t = z_0 + \gamma^z t + u_t^z,$$

i.e., the log of the variable follows a deterministic linear trend. As Eichenbaum and Hansen show, consistent estimation is possible if z_0, γ^z, and Ψ are jointly estimated.

The theoretical framework of Andrews and McDermott (1995) offers an alternative to de-trending in the presence of deterministic trends. Using triangular-array rather than traditional sequential asymptotic theory, Andrews and McDermott establish that consistent estimation is possible if the deterministic trend of the data has a particular structure relative to the economic model. Under such circumstances, model parameters and the asymptotic variance-covariance matrix can be estimated with the same procedures as in the case of strictly stationary regressors described above. The framework of Andrews and McDermott is convenient since it allows for a more general specification of the trend and is therefore applied here.

2.3 Interpretation of the MM Error

The MM error, u_t, measures the discrepancy between observed and model predicted values. In a deterministic model as the one introduced in Section 3, the error may be due to three aspects: (i) while the model is deterministic, real world data are generated by an unknown stochastic process and u_t reflects stochastic shocks, (ii) real world data are measured with error and u_t reflects this measurement error and (iii) u_t reflects specification error.

The issue of missing intrinsic stochastic components in the economic model is addressed here by first filtering observed time series of data using the Hodrick-Prescott procedure to decompose observed data z_t into a cyclical component r_t and a trend component τ_t (Hodrick and Prescott 1997). The discrepancy functions u_t are described using the deterministic components of the time series, τ_t, that reflect the smooth growth component of aggregate data.[7]

Let $\{Z_t\}_{t=1}^{T}$ be the observed time series of an aggregate economic variable, e.g., GDP and let $z_t = \ln(Z_t)$. The Hodrick-Prescott filter decomposes z_t into r_t and τ_t by solving the following programming problem

$$\min_{\{\tau_t\}_{t=1}^{T}} \left\{ \sum_{t=2}^{T} (z_t - \tau_t)^2 + \lambda \sum_{t=2}^{T} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right]^2 \right\}$$

for some predetermined parameter λ. For $\lambda \to \infty$, $\tau_t \to \tau_0 + \gamma t$ which is the least squares fit of a liner trend model. Since z_t is defined here as the log of the original variable, $\lambda \to \infty$ results in exponential growth of the trend component of the original variable Z_t. As Hodrick and Prescott point out, the linear trend specification is not an appropriate description of the data since the growth component varies "smoothly" over time. This feature of actual trends corresponds to the features of simulated trends of the model presented in Section 3. The appropriate λ-value for annual data recommended in the literature is 100.

To the extent that the de-trending procedure returns the "true" value of the deterministic component of the economic variable z_t of interest, the remaining interpretation for the MM error, u_t, is as specification error. However, there might be significant measurement errors of the original observed values of trending variables, Z_t. Let $Z_t^* = Z_t \exp \epsilon_t$ be the measured variable and let ϵ_t be the measurement error with the property that $\mathbb{E}\epsilon_t = 0$. As shown by King and Rebelo

[7] Note that this approach is just opposite to conventional procedures in the RBC literature where the cyclical component of the data is used for inference.

(1993), the solution of the above non-linear programming problem is a linear lag polynomial $\tau_t = (1 - h(L))z_t$, where $h(L)$ is the lag polynomial. Therefore, since the log of the measured variable is given by $z_t^* = z_t + \epsilon_t$, the measurement error also enters the de-trended variable linearly.[8]

In the presence of linear measurement error, the MM error writes as

$$u_t(\Psi) = y_t^* - h(\Psi) = y_t + \epsilon_t^y - f(y_t + \epsilon_t^y, x_t + \epsilon_t^x, \Psi) + \mu_t$$

Here, ϵ_t^y and ϵ_t^x are $r \times 1$ vectors of measurement error and μ_t is an $r \times 1$ vector of specification errors as before.

The presence of measurement error is problematic since under the assumption that $\mathbb{E}\epsilon_t^y = \mathbb{E}\epsilon_t^x = \mathbb{E}\mu_t = 0$, that is, under the assumption that measurement and specification errors are on average zero, the expected value of the MM error, $\mathbb{E}u_t$, may no longer be zero at Ψ^0.

For the economic model introduced in Section 3, equation f is, however, linear in y_t and x_t, hence

$$u_t(\Psi) = y_t^* - h(\Psi) = y_t + \epsilon_t^y - A(\Psi)y_t + B(\Psi)x_t + \mu_t + A(\Psi)\epsilon_t^y + B(\Psi)\epsilon_t^x$$

for some matrices $A(\Psi)$ and $B(\Psi)$. Therefore, the framework considered in this analysis allows for an interpretation of the error terms as linear specification error and as linear measurement error.

3 The Overlapping Generations Model

The AK-OLG model used to illustrate the above calibration procedure is a variant of the model used in Börsch-Supan, Ludwig, and Winter (2004). It is a multi-country extension of the standard OLG model by Auerbach and Kotlikoff (1987) which is augmented with realistic demographic data across these countries. The model has three building blocks: a demographic projection, a stylized pension system, and a macroeconomic overlapping generations model to calculate the general equilibrium of internationally linked economies. The following subsections contain a detailed description of all these elements. Readers familiar with large-scale multi-country AK-OLG models may skip Subsections 3.1 through 3.3.

3.1 The Demographic Model

Detailed demographic projections form the background of the analysis. Demography is taken as exogenous and represents the main driving force of the simulation

[8] Moreover, measurement error might be induced by the de-trending procedure itself, see, e.g., Conova (1998).

model.[9] In each country i, the size of population of age j in period t, $N_{t,j,i}$, is given recursively by

$$N_{t,j,i} = \begin{cases} \sum_{j=15}^{50} f_{t-1,j,i} N_{t-1,j,i} & \text{for } j = 0 \\ N_{t-1,j-1,i}(s_{t-1,j-1,i} + m_{t-1,j-1,i}) & \text{for } j > 0, \end{cases}$$

where $s_{t,j,i}$ denotes the age-specific conditional survival rate, $m_{t,j,i}$ the net migration ratio, and $f_{t,j,i}$ the age-specific fertility rate.

Individuals in the model economies enter economic life at the age of 20 which is denoted by $a = 1$. The maximum age as implied by the demographic projections is 104 years. Accordingly the maximum economic age, denoted by Z, is 85. To simplify calculations of the economic model, it is assumed that all migration takes place at the initial age of 20. This simplifying assumption allows to treat all "newborns" - immigrants and natives - in the economic model alike, see below.[10]

3.2 The Pension Model

Each region i is assumed to have a two-tier pension system. The first tier represents a conventional public pay-as-you-go (PAYG) system characterized by a country-specific contribution and replacement rate. More precisely, for each region i, the exogenous policy variable is the time-specific gross replacement rate, $\gamma_{t,i}$, defined as the ratio of average gross pension to average gross wage income at time t. The budget of the PAYG pension system is balanced at any time t and determines the contribution rate, $\tau_{t,i}$, by

$$\tau_{t,i} \sum_{a=1}^{Z} w_{t,a,i}^{g} l_{t,a,i}^{d} N_{t,a,i} = \sum_{a=1}^{Z} p_{t,a,i}(1 - l_{t,a,i}^{d}) N_{t,a,i}, \tag{20}$$

where pension benefits $p_{t,a,i}$ of a household of age a in time period t are calculated by

$$p_{t,a,i} = \gamma_{t,i} \lambda_{t,a,i} w_{t,a,i}^{g}$$

On the revenue side, $w_{t,a,i}^{g}$ denotes age-specific gross wages. Net wages are given by $w_{t,a,i}^{n} = w_{t,a,i}^{g}(1 - 0.5\tau_{t,i})$ under the assumption that half of contributions are paid by the employee and the other half by the employer. This latter half will be taken into account when firms maximize profits. $l_{t,a,i}^{d}$ denotes age specific labor supply shares resulting from optimal household decisions. The use of superscript d will be explained below.

[9] Assuming exogenous demographic processes is of course a simplifying assumption since, in the long run, neither fertility nor mortality and of course not migration is exogenous to economic growth.

[10] Both groups, newborns and immigrants, enter the economic model with zero assets. Furthermore, there are no skill differences between the two groups as analyzed by, e.g., Razin and Sadka (1999) and Storesletten (2000).

On the benefit side of the budget equation, pensions are defined by the general replacement rate and by a "point system" that credits $\lambda_{t,a,i}$ times the gross wage earned at age a. This is an approximation to the actual computation of pension benefits. Benefits are not taxed and interactions with other social protection systems are ignored.

3.3 The Macroeconomic Model

The two core elements of the macroeconomic general equilibrium model are the production and the household sector. They are presented separately here, although they are linked through several channels, in particular through the household's labor supply and savings decisions. The production sector in each country consists of a representative firm that uses a Cobb-Douglas production function given by

$$Y_{t,i} = F(\Omega_i, K_{t,i}, L_{t,i}) = \Omega_i K_{t,i}^{\alpha_i} L_{t,i}^{1-\alpha_i}, \tag{21}$$

where $K_{t,i}$ denotes the capital stock and $L_{t,i}$ aggregate labor input of country i at time t.[11] Labor supply is measured in efficiency units and α_i denotes the capital share.

Production efficiency of a household of age a at time t in country i has a factorial structure with three elements, relating to age, time and country. On the micro level, where households are distinguished by their age, labor productivity changes over the life-cycle according to age-specific productivity parameters ϵ_a. Hence, the age-specific gross wage is $w^g_{t,a,i}\epsilon_a$ and the aggregate labor supply is $L_{t,i} = \sum_{a=1}^{Z} \epsilon_a l_{t,a,i} N_{t,a,i}$, where $l_{t,a,i}$ denotes a single household's labor supply. Second, aggregate and individual labor supply ($L_{t,i}$ and $l_{t,a,i}$) are measured in efficiency units relative to a time endowment $E_{t,i}$. Age specific labor supply which corresponds to what is observed in the data is therefore given by $L^d_{t,a,i} = l_{t,a,i}N_{t,a,i}/E_{t,i}$. Superscript d is henceforth used to denote "*d*etrended" effective labor supply. The time endowment increases over time according to

$$E_{t,i} = E_{0,i} \exp(g_i t). \tag{22}$$

This "growth in time endowment" specification is equivalent to the standard labor augmenting technological change specification for the production sector and has useful properties for the specification of the household sector, see below. Third, Ω_i is the technology level of country i which is held constant over time.

In this version of the model, adjustment costs in the firm sector are not considered. Hence, profit maximization is static and the only constraint to firm

[11] The literature often examines the more general case of a CES production function (Altig et al. 2001). Estimation of the elasticity of substitution between capital and labor however results in a coefficient close to one and therefore the simpler case of a Cobb-Douglas production function is used here.

maximization is given by the capital accumulation condition

$$K_{t+1,i} = K_{t,i} + I_{t,i} - D_{t,i} = (1 - \delta_i)K_{t,i} + I_{t,i}, \tag{23}$$

where $I_{t,i}$ is gross investment, $D_{t,i}$ is depreciation and δ_i is the country-specific depreciation rate. The first-order conditions resulting from profit maximization give standard expressions for equilibrium wages

$$w^g_{t,i}(1 + 0.5\tau_{t,i}) = (1 - \alpha_i)\frac{Y_{t,i}}{L_{t,i}} \tag{24}$$

and interest rates

$$r_{t,i} = \alpha_i \frac{Y_{t,i}}{K_{t,i}} - \delta_i, \tag{25}$$

In order to determine aggregate consumption, savings and wealth, optimal household behavior derived from inter-temporal utility maximization is considered next. By choosing an optimal consumption path, each cohort born in time period t maximizes at any point in time $t + a$ and age a the sum of discounted future utility. The within-period utility function exhibits constant relative risk aversion, and preferences are additive and separable over time. Cohort t's maximization problem at $a = 1$ is given by

$$\max_{\{C_{t,a,i}, l_{t,a,i}\}_{a=1}^{Z}} \sum_{a=1}^{Z} \beta_i^{a-1} \pi_{t,a,i} U\left(C_{t,a,i}, l_{t,a,i}\right), \tag{26}$$

where β_i is the pure time discount factor. In addition to pure discounting, households discount future utility with their unconditional survival probability, $\pi_{t,a,i} = \prod_{j=1}^{a} s_{t,j-1,i}$. $C_{t,a,i}$ denotes consumption. Remember that a single household's labor supply, $l_{t,a,i}$, is measured in efficiency units relative to time endowment, $E_{t,i}$.

It is assumed that the period specific utility function is of the standard CES form given by

$$U\left(C_{t,a,i}, L_{t,a,i}\right) = \frac{1}{1-\theta_i}\left(\left(\left[\phi_i C_{t,a,i}^{-\gamma_i} + (1-\phi_i)(E_{t+a} - l_{t,a,i})^{-\gamma_i}\right]^{-\frac{1}{\gamma_i}}\right)^{1-\theta_i} - 1\right).$$

θ_i is the coefficient of relative risk aversion, $\phi_{a,i}$ the consumption share parameter, i.e. the weight of consumption relative to leisure in household's utility and $\xi_i = 1/(1 + \gamma_i)$ the intra-temporal substitution elasticity between consumption and leisure.[12]

[12] In contrast to the use of a Cobb-Douglas utility function, the more general CES utility function is used here. While the elasticity of substitution between capital and labor is close to one, which justifies the use of a Cobb-Douglas production technology in equation 21, compare footnote 11, there is a large agreement in the literature that the elasticity of substitution between consumption and leisure is below one (Altig et al. 2001).

Denoting total wealth by $A_{t,a,i}$, maximization of the household's intertemporal utility is subject to a dynamic budget constraint given by

$$A_{t,a+1,i} = \frac{1}{s_{t,a,i}} \left(A_{t,a,i}(1 + r_{t+a+1,i}) + l_{t,a,i} w^n_{t,a,i} + (E_{t+a} - l_{t,a,i}) p_{t,a,i} - C_{t,a,i} \right). \tag{27}$$

The term $1/s_{t,a,i}$ reflects the assumption of a perfect annuity market (Yaari 1965). This assumption is made to simplify computations and does not affect aggregate outcomes much. Income consists of asset income, net wages, and pensions. Maximization is also subject to the constraint that leisure may not exceed time endowment (and may not be negative)

$$0 \leq l_{t,a,i} \leq E_{t+a}. \tag{28}$$

The solution to the intertemporal optimization problem is characterized by two first-order conditions. The inter-temporal Euler equation describes the consumption growth rate of each household given by

$$C_{t,a+1,i} = C_{t,a,i} \left(\beta_i (1 + r_{t+a+1,i}) \frac{v_{t,a+1,i}}{v_{t,a,i}} \right)^{1/\theta_i}, \tag{29}$$

where $v_{t,a,i} = (\phi_i + (1 - \phi_i) clr_{t,a,i}^{-\gamma_i})^{-(1+\gamma-\theta)/\gamma}$. $clr_{t,a,i}$ is the consumption-leisure ratio defined by the intra-temporal Euler equation which relates current period consumption to current period leisure choice by

$$E_{t+a} - l_{t,a,i} = \left(\frac{1 - \phi_i}{\phi_i} \frac{1}{w^n_{t,a,i} + \mu_{t,a,i} - p_{t,a,i}} \right)^{1/(1+\gamma_i)} C_{t,a,i} = clr_{t,a,i} C_{t,a,i}, \tag{30}$$

where $\mu_{t,a,i} \geq 0$ is the shadow value of leisure. The "growth in time endowment" specification of the production function insures that steady state labor force participation is constant even if the elasticity of substitution between consumption and leisure is not equal one, i.e. if $\gamma_i \neq 0$ (Auerbach and Kotlikoff 1987; Altig et al. 2001).

The dynamic general equilibrium of the model economy is defined sequentially.[13]

Definition 1: A competitive equilibrium of the economy is defined as a sequence of disaggregate variables, $\{C_{t,a,i}, l_{t,a,i}, A_{t,a,i}\}$, aggregate variables, $\{C_{t,i}, L_{t,i}, K_{t,i}\}$, wage rates, $\{w_{t,i}\}$ in each country i and a common world interest rate, $\{r_t\}$ such that

[13] The definition of equilibrium as sequential coincides with the computational solution method (Ludwig 2004). It can be numerically computed since the model economy converges to a steady state and becomes a well-behaved system with a small number of equations.

- The allocations are feasible, i.e.

$$
\begin{aligned}
& Y_{t,i} + r_t F_{t,i} = S^n_{t,i} + C_{t,i} + D_{t,i} = S^g_{t,i} + C_{t,i} \\
= & \sum_{a=1}^{Z} (s_{t-a,a,i} A_{t+1-a,a+1,i} - A_{t-a,a,i}) N_{t,a,i} + \sum_{a=1}^{Z} C_{t-a,a,i} N_{t,a,i} + \delta_i K_{t,i},
\end{aligned}
$$

  where $F_{t,i}$ is the amount of foreign assets and $D_{t,i}$ is depreciation of capital and $S^n_{t,i}(S^g_{t,i})$ is net (gross) savings.

- Factor prices equal their marginal productivities as given in equations 24 and 25.

- Firms and households behave optimally, i.e., firms maximize profits subject to the capital accumulation constraint given in equation 23 and households maximize life-time utility given in equation 26 subject to the constraints in equations 27 through 28.

- All markets clear. Market clearing on national markets requires that

$$
\begin{aligned}
S^n_{t,i} &= \sum_{a=1}^{Z} S^n_{t-a,a,i} N_{t-a,a,i}, \quad & C_{t,i} &= \sum_{a=1}^{Z} C_{t-a,a,i} N_{t-a,a,i} \\
A_{t,i} &= \sum_{a=1}^{Z} A_{t-a,a,i} N_{t-a,a,i} \quad & L_{t,i} &= \sum_{a=1}^{Z} l_{t-a,a,i} N_{t-a,a,i}.
\end{aligned}
$$

  Market clearing on the international capital market and the assumption of perfect capital mobility across regions requires that the rate of return on financial investment is equalized across all countries,

$$
r_{t,i} = r_t, \tag{31}
$$

  and that the sum of all foreign assets, defined as the difference between home assets and the home capital stock, $F_{t,i} = A_{t,i} - K_{t,i}$, across all world regions equals zero, i.e.

$$
\sum_{i=1}^{R} F_{t,i} = 0,
$$

  where R is the total number of regions.

The time line of the model has four periods: a phase-in period, $t = -T^S, ..., 0$, a calibration period (1960-2003), $t = 1, ..., T$, a projection period (2004-2100), $t = T + 1, ..., T^P$ and a phase-out period, $t = T^P + 1, ..., T^E$, that lasts until 2300 when the model reaches a final steady state.

3.4 Moment Conditions

The total set of structural model parameters can be collected in the following vectors

$$\begin{aligned}\text{Production Sector: } \Psi^{PS} &= [\{\delta_i\}_{i=1}^R, \{\alpha_i\}_{i=1}^R, \{g_i\}_{i=1}^R, \{\Omega_i\}_{i=1}^R]' \\ \text{Household Sector: } \Psi^{HS} &= [\{\beta_i\}_{i=1}^R, \{\theta_i\}_{i=1}^R, \{\xi_i\}_{i=1}^R, \{\phi_i\}_{i=1}^R]'.\end{aligned}$$

However, not all of these parameters will be estimated by matching of moments. Since the open economy version of the model only serves as an illustration of the additional effects of openness, see below, the following simplifying assumptions are imposed:

$$\begin{aligned}&\delta_i = \delta_1; \alpha_i = \alpha_1; g_i = g \ \forall i \text{ and} \\ &\beta_i = \beta_1; \theta_i = \theta_1; \xi_i = \xi_1 \ \forall i.\end{aligned}$$

In other words, most of the parameters are estimated only for country $i = 1$.

In addition, a subset, Ψ^p, of the remaining calibration parameters are regarded as predetermined (i.e., as fixed by reference to other studies). Specifically, the elasticity parameters $1/\theta_1$ and ξ_1 are treated as predetermined since estimated values of these parameters would be outside ranges regarded as reasonable in the literature.

To summarize, predetermined parameters, Ψ^p, and estimated (free) parameters, Ψ^e, are given as follows:

$$\begin{aligned}&\Psi^p = [\theta_1, \xi_1]' \\ &\Psi^e = [\delta_1, \alpha_1, g_1, \{\Omega_i\}_{i=1}^R, \beta_1, \{\phi_i\}_{i=1}^R]'\end{aligned}$$

According to these assumptions, only the structural model parameters Ω_i and ϕ_i vary across countries. These parameters determine the effective “size” of each country in terms of technology levels (aggregate output, GDP) and in terms of the size of the aggregate labor force.

Remark Despite simplification, there is also a deeper role for the restrictions imposed in the open economy version of the simulation model that is due to an inconsistency between capital stock data and theoretical relationships of the above model. The market clearing condition in the open economy version of the model, equation 31, and the “no arbitrage” rule between financial and physical investment, equation 25, imply

$$\frac{Y_{t,j}}{K_{t,j}} = \frac{\alpha_i \frac{Y_{t,i}}{K_{t,i}} - \delta_i - \delta_j}{\alpha_j} \qquad i \neq j$$

a restriction that may not hold. Augmenting the simulation model with adjustment costs on physical capital investment is unlikely to solve this inconsistency and it could only be reasonably addressed by a model with additional components, e.g., with some market imperfection on the international capital market. Under the assumptions made here,

$$\frac{Y_{t,j}}{K_{t,j}} = \frac{Y_{t,i}}{K_{t,i}} \quad i \neq j.$$

This restriction is exploited below for the estimation of Ω_j, for $j > 1$.

3.4.1 Moment Conditions Underlying the Estimates of Ψ^e

Moment conditions for estimation of the structural model parameters Ψ^e follow directly from the above relationships of the theoretical model. Notice that lower case letters denote the log of the HP-filtered data. Recall that the estimation framework builds on the theoretical results established by Andrews and McDermott (1995) and therefore allows estimation using trending data. Also recall that the error terms, u_t may consist of two components, specification and measurement error, that both enter the logs of the HP-filtered data linearly.

From equation 23, δ_1 is estimated by

$$\mathbb{E}\left[d_{t,1} - k_{t,1} - \ln \delta_1\right] = 0,$$

and α_1 by transforming equation 24 as

$$\mathbb{E}\left[w_{t,1} + y_{t,1} - l_{t,1} - \ln(1-\alpha_1)\right] \;\; = \;\; 0.$$

The moment conditions underlying the estimates of Ω_i, the levels of total factor productivity, are derived from rewriting the production function, equation 21, in logs

$$\mathbb{E}\left[y_{t,i} - \ln \Omega_i - \alpha_i k_{t,i} - (1-\alpha_i)(l_{t,i} + g_i t)\right] \;\; = \;\; 0 \; \forall i = 1, ..., R.$$

The moment condition underlying the estimate g_1, the trend growth rate of efficiency units, is derived by taking first differences of the above equation as

$$\mathbb{E}\left[\gamma^Y_{t,1} - \alpha_1 \gamma^K_{t,1} - (1-\alpha_1)(\gamma^L_{t,1} + g_1)\right] \;\; = \;\; 0.$$

Since no closed form solution exists, estimation of structural model parameters of the household sector requires simulation. While the above moment conditions for the production sector imply stationarity of the MM error u_t at $\Psi^{e,0}$, this may not be the case for the household sector. For instance, as shown below, the endogenous labor supply model fails to replicate the growth rate of actual labor supply. Matching simulated to actual labor supply on average would then result in a non-stationary MM error even at $\Psi^{e,0}$. To address this, suitable normalization is required.

The moment condition underlying the estimate of the discount factor, β_1, is by matching the simulated to the actual average capital output ratio,

$$\mathbb{E}\left[k_{t,1} - y_{t,1} - \left\{\ln\left(\sum_{t,a}^{Z} H^s_{t-a,a,1}(\Psi, X) N_{t-a,a,1}\right) - y^s_{t,1}\right\}\right] = 0, \qquad (32)$$

where $H^s_{t,a,1}$ are simulated age-specific holdings of home assets by households of age a living in country 1 and $y^s_{t,1}$ is simulated output of country 1. Normalization by output insures stationarity of u_t at $\Psi^{e,0}$ if the model fails to match growth rates.

Identification of ϕ_i is by similar conditions on labor supply. Stationarity of u_t is achieved by deterministically de-trending. The moment conditions are accordingly given by

$$\mathbb{E}\left[l_{t,i} - \gamma_i^L t - \left\{\ln\left(\sum_{a=1}^{Z} \frac{l^s_{t-a,a,i}(\Psi, X) N_{t-a,a,i}}{E_{t,i}}\right) - \gamma_i^{L,s} t\right\}\right] = 0 \ \forall i = 1, ..., R. \qquad (33)$$

Division by $E_{t,i}$ is necessary since individual simulated labor supply is measured in efficiency units, see Section 3. Growth rates of labor supply, γ_i^L, are elements of Ψ^q, see below.

3.4.2 The parameters Ψ^q

Testing of the model within the calibration framework of Section 2 requires specification of the additional parameter vector Ψ^q. In the RBC literature, an obvious choice for Ψ^q are second moments, e.g., variance ratios of consumption to output. In the context of a deterministic model, this approach is not particularly meaningful. The basic idea of measuring variances and covariances - as being summary statistics that provide information on the time paths of variables - can however be nicely mapped into the AK-OLG framework where the statistics of interest are the relationships between the dynamics of aggregate variables and the dynamics of demographic change.

Figure 1 shows the time paths of the saving rate (solid line, left scale) and demographic measures such as the working age population ratio in Panel (a) and the old age dependency ratio in Panel (b) (dashed-dotted lines, right scale). The working age population ratio is defined as the ratio of the population in prime work age (aged 15 to 64) to total population and the old age dependency ratio is defined as the ratio of the old age population (aged 65 and older) to the working age population. All variables are shown as deviations from their deterministic trends. The graphs illustrate the positive relationship between the working age population ratio and the saving rate observed in the data and the strong negative relationship between the old-age dependency ratio and the saving rate. It is convenient to express such relationships in terms of correlations between the demographic measures and the macroeconomic variables of interest.

The figure also shows the predicted (and de-trended) saving rate for the open economy version of the model (dashed line), also see below. The predicted de-trended saving rate tracks the actual de-trended saving rate quite well with an exception being the period 1985-1995 where the decrease of the saving rate (relative to the trend) is under-predicted. Since the correlation statistic of two variables x and y normalizes the covariance by the standard deviations of both variables, this deviation would not be reflected in the correlation statistic. It is therefore convenient to express this additional information on the variation of the variables over the sample period in terms of the standard deviation of the de-trended variable of interest.

Furthermore, the simulation model may fail to match growth rates or levels of variables not used for estimation of Ψ^e. One way to summarize this is to look at the deviations of predicted growth rates of capital and labor supply.

These considerations motivate the definition of Ψ^q as

$$\begin{aligned}
\Psi^q &= [\gamma^K, \{\gamma^L\}_{i=1}^R, \sigma(x), \rho(x,z)]' \\
z &= WAPR, OADR \\
x &= \frac{K}{Y}, \frac{S}{Y} \quad \text{for the closed economy version of the model} \\
x &= \frac{K}{Y}, \frac{S}{Y}, \frac{I}{Y} \quad \text{for the open economy version of the model,}
\end{aligned}$$

where $WAPR$ and $OADR$ denote the working age population ratio and the old-age dependency ratio, respectively. $\sigma(x)$ denotes the standard deviation of variable x and $\rho(x,z)$ denotes the correlation coefficient between variables x and z.

The additional moment conditions used to estimate Ψ^q are therefore given by

$$\mathbb{E}\left[w_t - \Psi^q\right] = 0 \tag{34}$$

for $w_t = [\gamma_t^K,\ \{\gamma_t^L\}_{i=1}^R,\ \sigma(x_t),\ \rho(x_t, z_t)]$ and x_t, z_t defined as before.

3.5 Data

Below, different model versions of the simulation model will be used, see Section 4. The analysis focuses mostly on the US. In addition, a two-country open economy version of the model will be simulated. The second country thereby represents a country aggregate of all OECD countries other than the US.

For the US, national income and product accounts (NIPA) data are used taken from the Bureau of Economic Analysis (BEA). While model simulation starts in 1950, the first ten years are discarded and structural model parameters are estimated using sample data for the years 1960-2003. Throughout, data for the entire economy are used. Since there is no real role for a government in the model, it is therefore implicitly assumed that the government is a substitute to the private sector. All real data are calculated using the GDP deflator.

The capital stock is defined as the sum of fixed capital held by the private and the public sector and private inventories. Depreciation is calculated to be consistent with the data on the capital stock and the investment flow satisfying the capital accumulation equation 23. Consumption is calculated as the sum of private consumption and government consumption. In the closed economy version of the model, no additional correction to the data by deducting consumption of imported goods and services is made. The reason for not doing this correction is that the model comparison in Section 4 would be flawed if different data sets were used in the open and closed economy scenarios. However, this also implies that actual data on investment and savings differ in the closed economy models, whereas simulated data on these variables are equal by definition of the closed economy. Finally, output is defined as the sum of investment and total consumption (including government consumption) which corresponds to actual GDP as observed in the data.

As a measure of aggregate gross wages, data on total compensation of employees is used which includes supplements to wages and salaries. Labor supply is measured as actual labor supply multiplied by an index for the total amount of hours worked. The wage rate is calculated as total wages divided by the weighted labor supply data.

The open economy version of the model focuses on OECD countries. Data on GDP and labor supply for these countries are taken from the World Development Indicators (World Bank 2003). Some minor adjustments are made to ensure consistency between the US data and the data used for the other model economies. This data is summed across all countries to obtain the data for the country aggregate "other OECD countries", see Section 4.

For sake of consistency between the demographic and the economic model, especially with regard to mortality rates that enter the household's objective function, demographic projections are explicitly calculated. They are based on the United Nations World Population Projections (United Nations 2002). The demographic model is calibrated such as to match the data. The resulting demographic data are taken as exogenous in the estimation exercises conducted in Section 4. Since the fit of the demographic model is good, results of errors in the imputation procedure on simulation outcomes are found to be low (results not shown).

Pension payments are calculated as the sum of the NIPA data on pension payments for old-age, survivors and disability insurance, railroad retirement, pension benefit guarantee and pension and disability insurance of veterans. The pension system's overall contribution rate is calculated by dividing pension payments through the data on wages and salary accruals. The pension system's net replacement is determined using the pension system's budget constraint in equation 20 for the exogenous labor supply model. Across all simulations, net replacement rates are held constant at the resulting level and contribution rates are endogenously calculated.[14] For the remaining countries, the public pension system's gross replacement rates are calculated using data from Palacios and Pallarès-Miralles (2000). Contribution rates provided in Blöndal and Scarpetta (1999) are used to calculate net replacement rates.

4 Results

In what follows, three different sub-models of the AK-OLG model of Section 3 are analyzed. Model I is an exogenous labor supply, closed economy model, hence $R = 1$ and $\phi_1 = \xi_1 = 1$. Model II is an endogenous labor supply, closed economy model and Model III is an endogenous labor supply, open economy model. In the open economy version, $R = 2$ countries (regions) will only be considered which simplifies computations. The second model region consists of all OECD countries other than the US. Table 1 summarizes these model properties.

4.1 First Results: The Role of Technology

As a first step, Model I (closed economy, exogenous labor supply) is analyzed in two versions. First, it is assumed that productivity follows the constant trend growth assumption of equation 22 and that total factor productivity (TFP) $\Omega_{i,t}$ is held constant over time. In slight abuse of notation relative to Section 3, a time subscript t is added to the TFP-Level here. However, the constant trend growth assumption is not the most reasonable description of actual technological change. Therefore, a second version is analyzed where the assumption that $\Omega_{t,i} = \Omega_i$ for $t < 1$ and $t > T$, is maintained, i.e., out of sample, the TFP level is held constant, but where, in sample, $\Omega_{t,i}$ is replaced with the actual "Solow-Residual" (equivalent) resulting from the growth regressions, $SR_{t,i}$, i.e., $\Omega_{t,i} = SR_{t,i}$ for $1 \leq t \leq T$. Notice that $SR_{t,i}$ is a stationary variable in this model which explains the above use of the word "equivalent". Feeding $SR_{t,i}$ explicitly into the simulation model allows to account for the effects of potential changes in aggregate productivity, like a productivity slowdown, that are ruled out by the constant growth assumption of $E_{t,i}$. The Solow-Residual (equivalent) is defined as

$$SR_{t,i} = \frac{Y_{t,i}}{K_{t,i}^{\alpha_i} L_{t,i}^{1-\alpha_i}}.$$

[14]This reversal in the procedure ensures that replacement rates continuously rise as implied by the exogenous labor supply scenario also in the endogenous labor supply scenario. This may not be the case if replacement rates were calculated endogenously in both scenarios holding contribution rates fixed.

Recall that $L_{t,i}$ is efficient labor which is trending over time.

Feeding the actual Solow-Residual, $SR_{t,i}$, into the model implies that output during the simulation period is given by

$$Y_{t,i}^s = SR_{t,i}(K_{t,i}^s)^{\alpha_i}(L_{t,i}^s)^{1-\alpha_i},$$

where $Y_{t,i}^s$, $K_{t,i}^s$ and $L_{t,i}^s$ denote simulated output, capital and labor, respectively. This also implies that simulated wages are given by

$$w_{t,i}^{g,s}(1+0.5\tau_{t,i}) = (1-\alpha_i)SR_{t,i}(K_{t,i}^s/L_{t,i}^s)^{\alpha_i}$$

and simulated interest rates by

$$r_{t,i}^s = \alpha_i SR_{t,i}(L_{t,i}^s/K_{t,i}^s)^{1-\alpha_i} - \delta_i.$$

The additional argument $SR_{t,i}$ hence affects the time paths of households labor and asset income and thereby alters their labor supply, consumption and savings decisions relative to the constant trend growth assumption.

Figure 2 shows results on actual and predicted output and the capital stock per efficient unit of labor, $Y_{t,1}/L_{t,1}$ and $K_{t,1}/L_{t,1}$, respectively, for the two versions of Model I. While the model fails to match the time paths of both variables, the "Solow-Residual" model version does a much better job, especially with regard to tracking the observed swings of the capital-output ratio. For this reason, the remainder of the analysis focuses on models where, in sample, the constant technology level is replaced with the actual Solow-Residual (equivalent).

4.2 Main Results: The Roles of Endogenous Labor Supply and Openness

4.2.1 Parameter Estimates of Ψ^e

Table 2 contains predetermined and estimated parameter values of the vector of structural model parameters $\Psi^c = [(\Psi^p)', (\Psi^e)']'$ for Models I through III. Values of predetermined parameters, Ψ^p, are chosen in accordance with the literature. The value of the elasticity parameter ξ corresponds to the value chosen by Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001). Values of estimated parameters, Ψ^e, are within ranges considered as reasonable in the literature. The point estimates of the discount factor, β, correspond to the value value of the discount rate of 0.011 estimated by Hurd (1989). Notice, however, that the estimated value depends on the value of the predetermined parameter θ, the coefficient of relative risk aversion. A higher (lower) θ-value implies a higher (lower) discount factor (results not shown).

Standard errors of the estimated parameters Ψ^e are based on the un-weighted, truncated kernel Hansen-Hodrick-White (HHW) estimator of $\widehat{S}_T$ given by

$$\widehat{S}_T = \sum_{i=-T+1}^{T-1} k(i)\widehat{C}_i$$

with

$$\widehat{C}_l = \frac{1}{T-J}\sum_{t=l}^{T} u_t(\widehat{\Psi^e}_T)u_{t-l}(\widehat{\Psi^e}_T)'$$

and with the Bartlett kernel defined as

$$k(i) = \begin{cases} \left(1 - \frac{|i|}{b}\right)^{\upsilon}, & 0 \leq |i/b| \leq 1 \\ 0, & |i/b| > 1. \end{cases}$$

for $\upsilon = 0$ and for a fixed bandwidth of $b = 4$ years (Hansen and Hodrick 1980; White 1984). Results obtained with the alternative Newey-West (NW) kernel estimator with $\upsilon = 1$ are similar (Newey and West 1987). The advantage of the HHW-Estimator over the NW-Estimator estimators is that it does use all the information in $\widehat{C}_t$ until the truncation point. The disadvantage is that positive definiteness of the resulting estimate of $\widehat{S}_T$ is not guaranteed. Here, this was not the case for $b = 4$. Consistency of $\widehat{S}_T$ requires that the truncation point, the bandwidth parameter b, approaches infinity at the appropriate rate as T goes to infinity (Andrews and Monahan 1992). Automatic selection criteria for the optimal bandwidth b that optimize asymptotic efficiency criteria have been developed by Andrews (1991) and Newey and West (1994). However, as discussed by Christiano and Den Haan (1996), neither of these procedures is entirely automatic since they require exogenous parameter selection at a different stage. Therefore, results obtained for a fixed bandwidth are reported here. The parameters are estimated with high precision, see Table 2.

4.2.2 Informal Model Evaluation

Figures 3 and Figures 4 summarize simulation results obtained for Models I-III if the Solow-Residual (equivalent) replaces the constant TFP level. As before, the solid lines represent the data and the dashed lines represent results for Model I (closed economy, exogenous labor supply). Simulation results for Model II (closed economy, endogenous labor supply) are represented by the dashed-dotted lines and results for Model III (open economy, endogenous labor supply) are represented by the dotted lines.

Results can be summarized as follows: First, the endogenous labor supply model fails to match the average growth rate of actual labor supply, see Panel c of Figure 3 depicting actual and predicted labor supply shares. Results on predicted labor supply shares between Models II and III are indistinguishable. As further shown in Table 3 below, the model at the same time overestimates the trend growth rate of labor supply in the second country. The failure of the model to match the data along the labor supply dimension is not related to the predetermined parameter ξ (results not shown). It can therefore be concluded that the above way of modelling labor supply is an imperfect, and to some extend surprisingly inflexible approximation of actual labor supply decisions.

Second, while Model I to some extent matches the timing of swings (but not their amplitudes) of the actual capital-output ratio, this is no longer the case for Models II and III prior to about 1980, see Panel (b) of Figure 3. Both, modelling endogenous labor supply and openness also "smoothes out" the variation of the capital-output ratio; see also Panel (a) of Figure 4. Third, Models II and III seem to track de-trended output a bit closer, see Panel (a) of Figure 3.

Fourth, Model I appears to lead the data by about ten years with respect to the fall of the saving rate observed in the early 80s and the subsequent rise observed in the 90s, see Panel (d) of Figure 4. The drop of the saving rate also appears too early in Model II, whereas for Model III the decline of savings appears at the same time as observed in the data, see also Figure 1. For both Models II and III predicted saving rates remain roughly constant throughout the 80s and 90s. Fifth, and in correspondence with these findings, Models II and III do a slightly better job in tracking the persistent increase in the consumption-output ratio, see Panel (b) of Figure 4. Finally, non of the models matches the time path of the investment ratio, see Panel (c) of Figure 4.

4.2.3 Formal Model Evaluation

Results on the moments of the data collected in Ψ^q and their simulated counterparts $h^q(\Psi^e)$ are shown in Table 3. These results more or less confirm the findings obtained in the graphical analysis. For instance, since all models fail to match the actual variation of the capital-output ratio, the predicted standard deviation of the de-trended capital output ratio is lower than in the data (and it decreases across models). All models replicate the positive (and significant) correlation between the capital-output ratio and the working age population ratio. The correlation between the old-age dependency ratio and the capital-output ratio is found to be insignificant in the data which is replicated by Model II (although with the wrong sign).

All models are found to replicate the sample variation of the saving rate. The correlation between the saving rate and the working age population ratio is found to be insignificant which is replicated by Models I (although with the wrong sign) and II but not by Model III (but with the correct sign). All models match the significant negative correlation between the saving rate and the old age dependency ratio.

Results of formal J-Tests are reported in the lower part of Table 3. J_7 is the J-Statistic based on the all moments relevant for Model I, hence the (7×1) vector $[\gamma^K,\ \sigma(x),\ \rho(x, WAPR),\ \rho(x, OADR)]'$ for $x = K/Y,\ S/Y$. Unsurprisingly, all models are rejected according to this criterion. The J_3-Statistic is based on all moments of the saving rate, that is, on the (3×1) vector $[\sigma(S/Y),\ \rho(S/Y, WAPR),\ \rho(S/Y, OADR)]'$. According to the findings of this statistical criterion, Models II and III cannot be rejected with regard to the moments of the actual saving rate at the 0.48 and the 0.08 level of significance, respectively.

5 Conclusions

This paper develops a systematic calibration procedure for large-scale Auerbach-Kotlikoff-OLG (AK-OLG) models in outside steady state situations. Structural model parameters are estimated by matching first moments of model predicted, in some cases simulated, values to long time series of aggregate data. It is found that the procedure works well and that resulting parameter values are within ranges considered as reasonable in the literature. As an illustration, three versions of a prototype AK-OLG model are evaluated using informal graphical analysis and by formal statistical criteria that complement the graphical analysis.

The illustrative AK-OLG model developed in this paper is an open economy AK-OLG model that features realistic demographic profiles. While it is well-suited for the questions addressed in this paper along these two dimensions, a number of aspects which have been regarded as important in the literature are missing: For example, the model does not account for bequest motives, within age group heterogeneity, idiosyncratic and/or aggregate uncertainties, human capital formation and a detailed representation of the government sector (Imrohoroglu, Imrohoroglu, and Joines 1995; Conesa and Krueger 1999; Altig, Auerbach, Kotlikoff, Smetters, and Walliser 2001; Krueger and Kubler 2003). Against this background, results derived from the model evaluation procedure must be tentative. They nevertheless allow the following insights: First, allowing the actual Solow-Residual resulting from growth regressions to enter the simulation model rather than assuming TFP to grow linearly at a constant rate significantly improves the performance of the exogenous labor supply version of the simulation model. Second, modelling endogenous labor supply decisions as resulting from pure life-time utility maximization over consumption and leisure fails to match the data. Third, the endogenous labor supply and open economy versions of the model are shown to match the saving rate quite well. Forth, all models fail to match the time paths of investment and consumption.

What explains these discrepancies between actual and simulated data? Certainly, a good proportion of the discrepancies may be due to the features missing in the model, and the failure of the model also reflects the inadequacy of the life-cycle theory of consumption and savings (Attanasio 1999). The above mentioned results point to three distinct but related aspects which may provide guidance for future model developments: First, the way in which technological progress is modelled is found to be important. This is not only important for the back-fitting implications but also for the analysis of future macroeconomic developments and of future public policy. Second, better models of the labor market are needed and third, improved ways of modelling the open economy and physical capital investment are required.

A fourth and related aspect that is not addressed in the above analysis is the role of capital depreciation. The constant depreciation rate assumption made above may explain why a model that is augmented with the actual Solow-Residual still fails to match a large proportion of the observed fluctuations of the capital-output ratio. The importance of both, non-constant technology and non-constant depreciation may also point into the direction of missing intrinsic model uncertainty that could be modelled by adding technology shocks and shocks to depreciation. Such extensions however imply a huge increase in the computational costs required to solve such models (Krueger and Kubler 2003).

A few final comments on the econometric methodology are in order. The econometric methodology applied here is with a classical statistical perspective. In other words, calibration parameters are regarded as an unknown but fixed number. The uncertainty reflected in the estimated variance-covariance matrix is due to sampling uncertainty. Apart from the values of predetermined model parameters that assumes degenerated priors in a Bayesian sense, prior information on model parameters is not incorporated. For the last decade, the RBC literature has seen numerous developments of Bayesian approaches to estimate and test dynamic macroeconomic models. Reviews are provided in Kim and Pagan (1995) and Canova and Ortega (1999). Bayesian methods regard parameter values themselves as random variables and express inference in statements of probability regarding their value. They are the standard procedure to combine uncertainty about prior distributions of parameter values with the uncertainty implied by the data.

In the context of the above application, Bayesian methods would "kill three birds with one stone": First, they do not require the artificial distinction between predetermined and estimated parameters made above. Second, they incorporate uncertainty over all model parameters and allow for use of prior knowledge on parameter values derived from other studies. Third, and finally, the literature more recently developed methods not only to compare models to the data but also to compare different sub-models. The Bayesian approach is attractive in this context since model uncertainty is handled in the same manner as any other uncertainty in the model even if models are not nested (Fernández-Villaverde and Rubio-Ramírez 2002). Embedding the above analysis in a Bayesian framework is subject to future research.

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Table 1: Properties of Models I-III

Property	Model I	Model II	Model III
Endogenous Labor Supply	No	Yes	Yes
Open Economy	No	No	Yes
Implication			
R	1	1	2
ϕ_1	1		
ξ_1	1		

Notes: This table summarizes properties of Models I-III and the implied restrictions on parameter values.

Table 2: Structural Model Parameters Ψ^c for Models I-III

Ψ^p	Model I	Model II	Model III
θ: coefficient of relative risk aversion	2	2	2
ξ: intra-temporal substitution elasticity	1	0.8	0.8
ϕ_1: consumption share parameter	1		
Ψ^e		Models I-III	
δ: depreciation rate		0.037	
		(0.002)	
α: capital share parameter		0.329	
		(0.004)	
g: growth rate		0.017	
		(0.002)	
Ω_1: technology level		0.077	
		(0.002)	
Ψ^e	Models I	Model II	Model III
Ω_2: technology level			0.062
			(0.004)
β: discount factor	0.991	0.996	0.989
	(0.004)	(0.005)	(0.005)
ϕ_1: consumption share parameter		0.608	0.610
		(0.009)	(0.009)
ϕ_2: consumption share parameter			0.570
			(0.007)

Notes: This table shows predetermined parameter values, Ψ^p, and estimated parameter values, Ψ^e, of the structural model parameters Ψ^c for Models I-III. Standard errors are calculated using the Hansen-Hodrick-White (HHW) estimator with bandwidth parameter $b = 4$.
Source: Own calculations, based on demographic projections of the United Nations (2002).

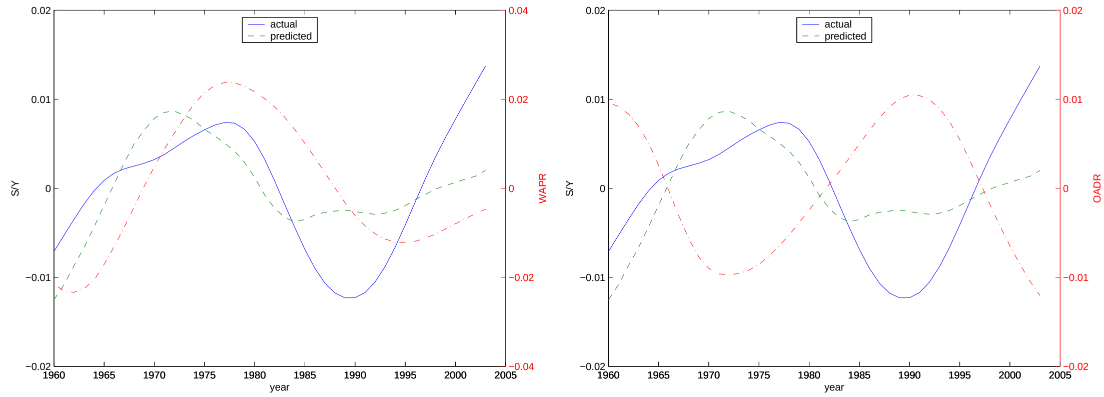
Table 3: Parameters Ψ^q, simulated values $h^q(\Psi^e)$ and J-Statistics for Models I-III

Parameter	Ψ^q	$h^q(\Psi^e)$		
	Data	Model I	Model II	Model III
γ^K	0.033	0.034	0.031	0.031
	(0.003)	(0.004)	(0.005)	(0.004)
γ_1^L	0.017		0.014	0.014
	(0.001)		(0.001)	(0.001)
γ_2^L	0.009			0.013
	(0.001)			(0.001)
$\sigma(K/Y)$	0.122	0.044	0.035	0.022
	(0.022)	(0.019)	(0.019)	(0.022)
$\rho(K/Y, WAPR)$	0.667	0.941	0.946	0.916
	(0.299)	(0.250)	(0.277)	(0.294)
$\rho(K/Y, OADR)$	0.154	-0.590	-0.291	-0.642
	(0.225)	(0.227)	(0.243)	(0.255)
$\sigma(S/Y)$	0.007	0.009	0.006	0.005
	(0.001)	(0.002)	(0.001)	(0.002)
$\rho(S/Y, WAPR)$	0.344	-0.311	0.150	0.641
	(0.256)	(0.203)	(0.230)	(0.237)
$\rho(S/Y, OADR)$	-0.900	-0.586	-0.717	-0.838
	(0.269)	(0.234)	(0.264)	(0.270)
$\sigma(I/Y)$	0.005			0.006
	(0.001)			(0.002)
$\rho(I/Y, WAPR)$	0.672			-0.481
	(0.320)			(0.315)
$\rho(I/Y, OADR)$	-0.385			-0.484
	(0.282)			(0.283)
J-Statistic				
J_7: Ψ^q elements of Model I		288.536	262.050	94.306
		[0.000]	[0.000]	[0.000]
J_3: S/Y		15.934	2.471	6.620
		[0.001]	[0.485]	[0.088]

Notes: The upper part of this table shows estimated values of the model parameters Ψ^q and their simulated counterparts $h^q(\Psi^e)$ for Models I-III. Standard errors are calculated using the Hansen-Hodrick-White (HHW) estimator with bandwidth parameter $b = 4$ and are reported in parentheses. The lower part shows results of two J-Statistics: J_7 is the J-Statistic based on the (7×1) vector $[\gamma^K, \sigma(x), \rho(x, WAPR), \rho(x, OADR)]'$ for $x = K/Y, S/Y$. J_3 is the J-Statistic based on the (3×1) vector $[\sigma(S/Y), \rho(S/Y, WAPR), \rho(S/Y, OADR)]'$. p-values are reported in brackets.
Source: Own calculations, based on demographic projections of the United Nations (2002).

Figure 1: Saving rates and population statistics (deviations from trend)

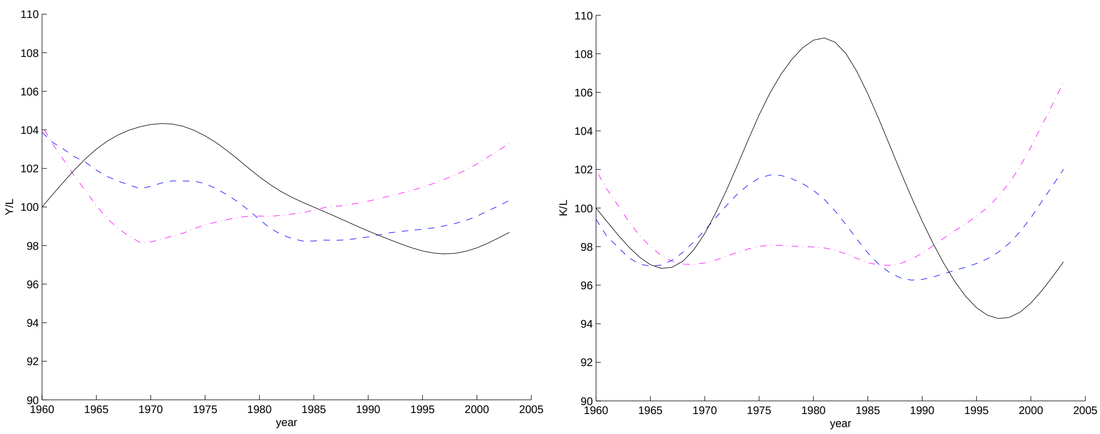
a. saving rate and working age population　　　b. saving rate and old age dependency



Notes: Each panel of this figure shows, on the left scale, actual (solid line) and predicted (Model III, dashed line) values of saving rates and population statistics (dashed-dotted line) on the right scale. Population statistics shown are the working age population ratio in Panel (a) and the old age dependency ratio in Panel (b). All series are shown as deviations from their deterministic trends for the period 1960-2003.
Source: Own calculations, based on demographic projections of the United Nations (2002).

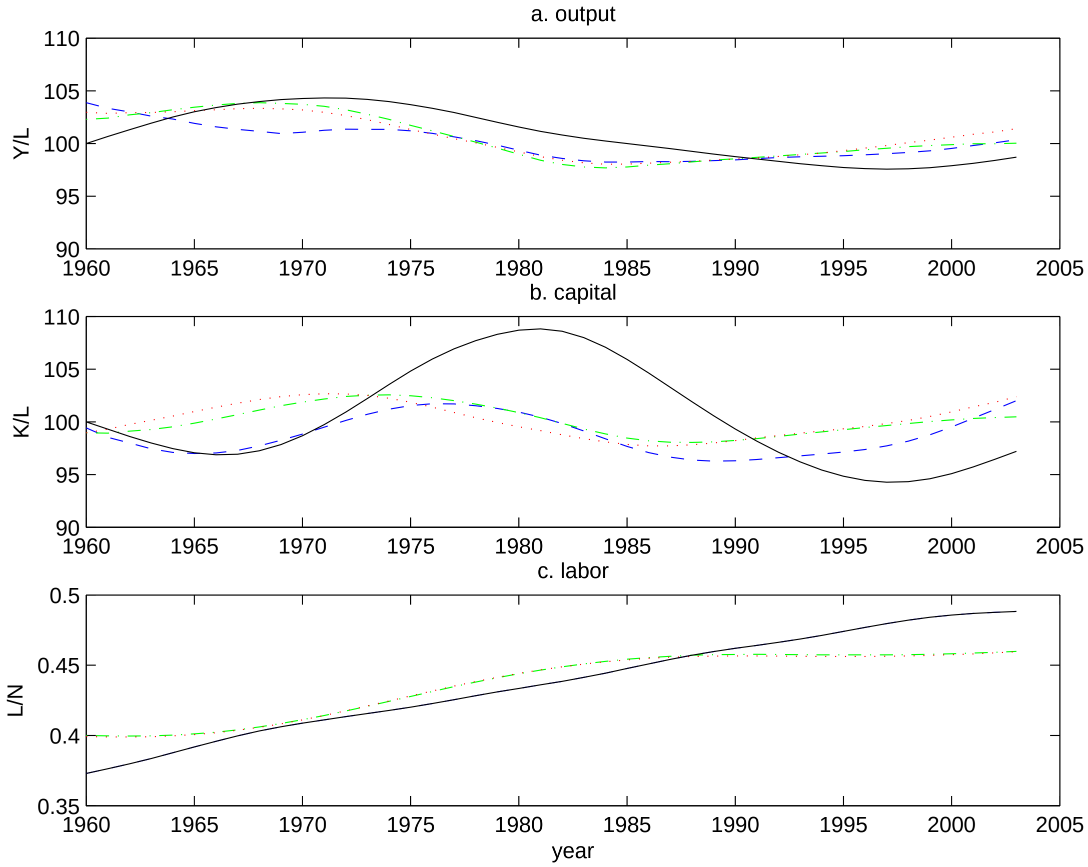
Figure 2: The role of technology: Output and capital stock per efficient unit of labor for Model I

a. output b. capital stock



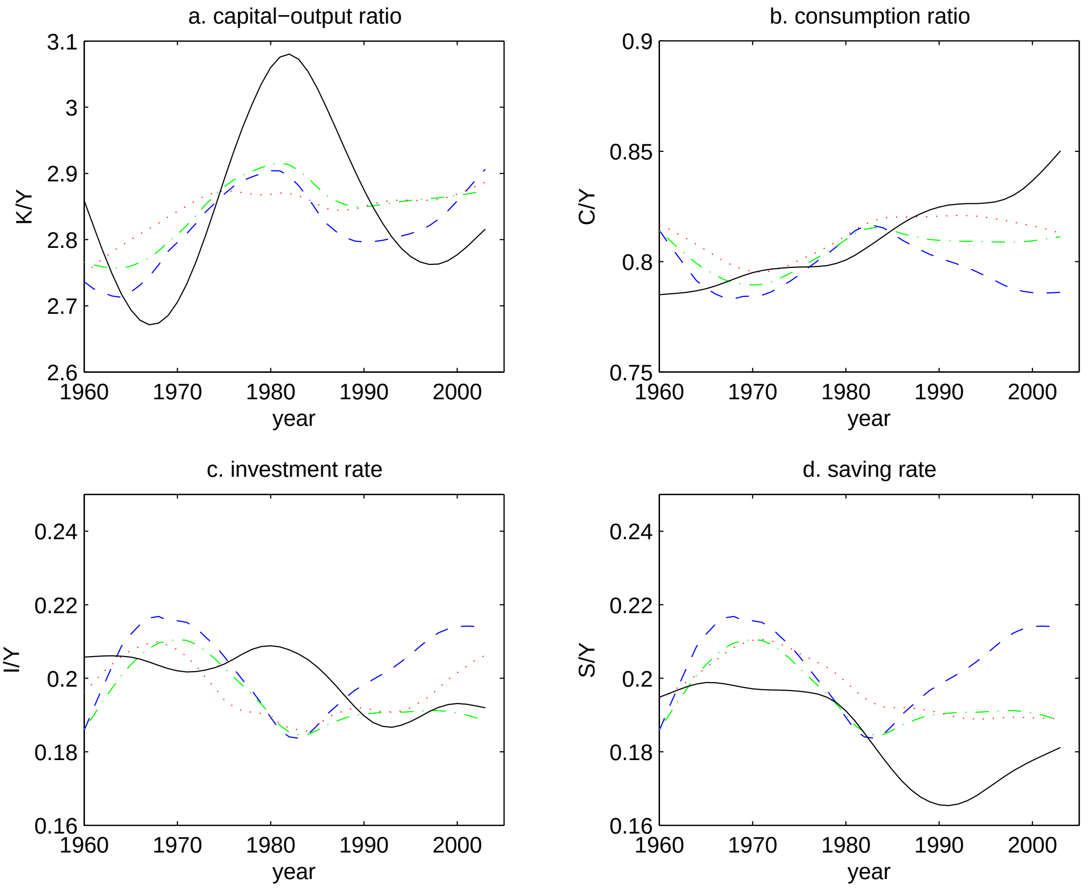
Notes: This graph shows actual values (solid line) and predicted (Model I) values (dashed dotted line: constant TFP, dashed line: Solow-Residual) of output and of the capital stock per efficient unit of labor.
Source: Own calculations, based on demographic projections of the United Nations (2002).

Figure 3: The roles of endogenous labor supply and openness: Output and capital stock per efficient unit of labor and labor supply for Models I, II and III



Notes: This graph shows actual (solid line) and predicted (Models I-III) values (dashed line: Model I, dashed-dotted line: Model II, dotted line: Model III) of output and the capital stock per efficient unit of labor and of labor supply.
Source: Own calculations, based on demographic projections of the United Nations (2002).

Figure 4: The roles of endogenous labor supply and openness: Capital stock, consumption investment and savings as percentage of GDP for Models I, II and III



Notes: This graph shows actual values (solid line) and predicted (Models I-III) values (dashed line: Model I, dashed-dotted line: Model II, dotted line: Model III) of the capital-output ratio, the consumption rate, the investment rate and the saving rate.
Source: Own calculations, based on demographic projections of the United Nations (2002).

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