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Note on the Stock-Wise utility function used in their option-value analysis

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Abstract

The option value of postponing retirement is the difference between the utility when retiring at the age that maximizes utility minus the utility when retiring now. This note shows that a utility function which depends only weakly on the value of leisure such as the utility function proposed by Stock and Wise (1990) and used in many applications (see Gruber and Wise, various issues) makes this difference flat relative to a more leisure-sensitive utility function. This explains the poor results observed in many European countries which have participated in the Gruber-Wise exercise where the option value was based on the Stock-Wise utility function and in which leisure (here: early retirement) is much more highly valued than in the United States.

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Note on the Stock-Wise utility function used in their option-value analysis

Axel Börsch-Supan, MEA&NBER

Stock and Wise (1990, in the sequel abbreviated as SW)[1] and the work inspired by this approach (e.g. Gruber and Wise, Börsch-Supan and Schnabel, various issues) use the following utility function to compute the option value of postponing retirement:

(1) V(y,p)=u(y)+k*u(p)

The parameter k>1 measures the weight which is given to unearned public pension (in the US: social security) income p relative to labor income y. It is interpreted as the disutility of working in order to earn labor income.

Income yields a consumption utility u(z) which maybe specified as z^{gamma}.

This brief note shows that this utility function is degenerated. Essentially, it prevents utility from leisure to outweigh utility from consumption.

A significant side effect of this degeneration is its implication for the adjustment of pension benefits when postponing retirement should be incentivized. While such adjustments are usually computed equating the PDVs of net pension income at different retirement ages, corresponding adjustments may also be computed equating the utility from work and leisure at different retirement ages. The SW utility function produces utility-based adjustment factors that are always lower than PDV-based actuarial adjustments, independent how large the disutility from working is. This is a counterintuitive result due to the degeneration.

The standard textbook log utility function

(2) U(c,l)=alfa*log(c)+(1-alfa)*log(l)

for consumption c and leisure l does not have this property.

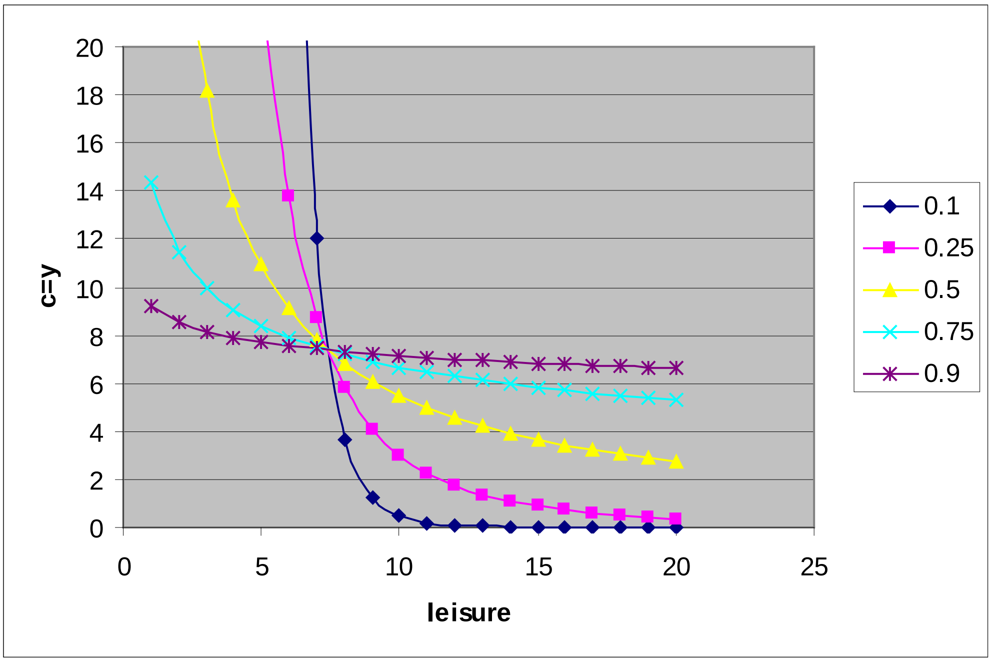
To show this, let's first map (y,p) space into the conventional textbook (c,l) space in a very abstract way. First, ignoring saving, consumption equals total income, c=y+p. Second, leisure time is, in first order, proportional to retirement income, l=const*p, when l is measured in years and p lifetime retirement income. Redefining k and the absolute value of V, we can therefore identify (1) with

(3) V'(c,l)=u(c)+k'*u(l).

Next, we draw the indifference curves in (c,l) space for both utility functions.

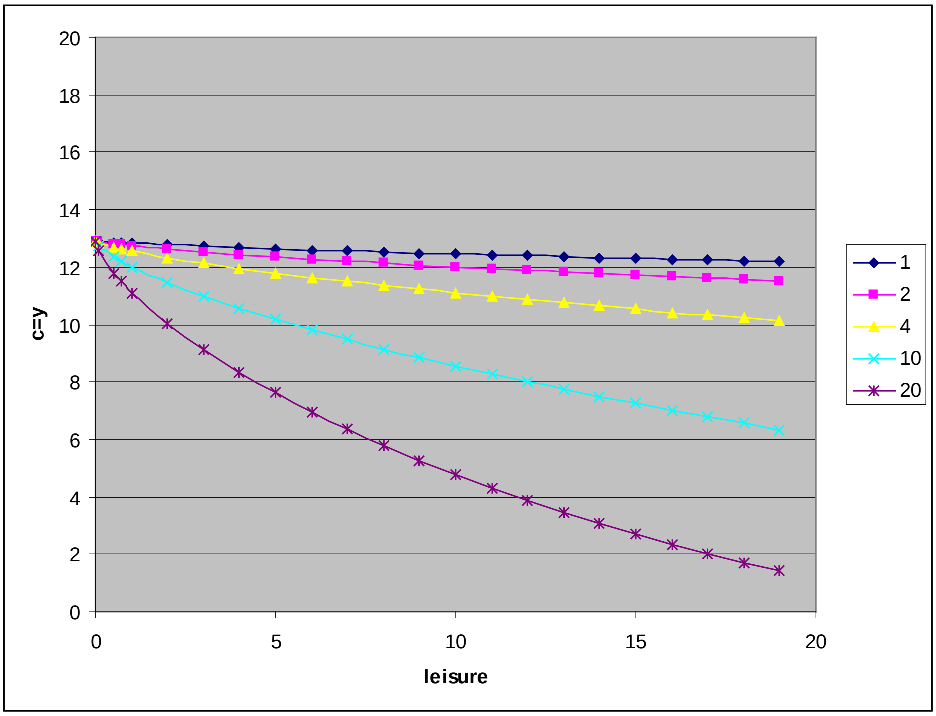
For the standard textbook log utility function (2), this yields the well-known indifference curves c = exp(V-(1-alfa)*log(l)):

[1] Stock, J.H., and Wise, D.A. (1990), The Pension Inducement to Retire: An Option Value Analysis, in: D.A. Wise (eds.) "Issues in the Economics of Aging", Chicago, 205-30.



These indifference curves fulfil the (Inada?) condition that u(x,z) goes to 0 as either x or z goes to zero; or, formulated differently, consumption has to grow very large in order to compensate for very little leisure time.

This condition does not hold for the SW utility function (3) as seen by the indifference curves $c = (U - k^*l^{gamma})^{(1/gamma)}$



Even if the work disutility parameter k is very high, the indifference curves are bound from above. Thus, when leisure becomes smaller and smaller, consumption (thus labor income) to compensate for this does not have to increase to large and larger amounts, violating the (Inada?) condition.

This anomaly has implications for utility-based adjustments to pension benefits when retirement is postponed. Adjustment factors are usually computed on an actuarial basis as follows. Let the PDV of net retirement benefits when retiring a year from now be

(4) $-b*y + D*p*(1+a)$

which represents this year's net income plus the pension income during the remaining life. b<1 is the contribution rate to the pension system, y labor income of the current year, p constant pension income from next year on including (1+ a), the adjustment factor for retiring next year rather than today. D is a constant comprising the length of retirement, survival probabilities, and discounting.

When retiring already this year, (4) becomes

(5) $p + D*p$

where pension benefits do not enjoy the actuarial incentive through (1+a).

Solving the condition (4)=(5) for (1+a) yields

(6) $(1+a) = 1 + 1/D + b/rD$

where r is the replacement rate p = r*y.

One may argue that the adjustment factor should not be based on an actuarial computation based on PDVs but on utility.[2] Hence, (1+a) should be computed to leave individuals indifferent (in a utility sense, not PDV sense) between retiring now or later.

Using the SW utility function (1), equations (4) through (6) become

(7) $(1-b)*y + k*D*p*(1+a)$
(8) $k*p \quad + k*D*p$
such that
(9) $(1+a) = 1 + 1/D + (b-1)/krD$

Note that (1+a) derived from equation (9) will always be smaller than (1+a) derived from equation (6), independent of the value of k since the last term of (9) is negative (b<1) while the last term in (6) is positive; see also the figure below. This is counterintuitive since a very high value of k ("abhorrence of labor") should require a very high value of (1+a) in order to incentivize individuals to work longer. This counterintuitive result is a reflection of the abnormal indifference curves characterizing the SW utility function.

The standard textbook log utility function (2) does not have that property. The equations equivalent to equations (4) through (6) are now:

(10) $alfa*log((1-b)*y+D*p*(1+a)) + (1-alfa)*log(D)$
(11) $alfa*log(\quad p+D*p \quad) + (1-alfa)*log(D+1)$

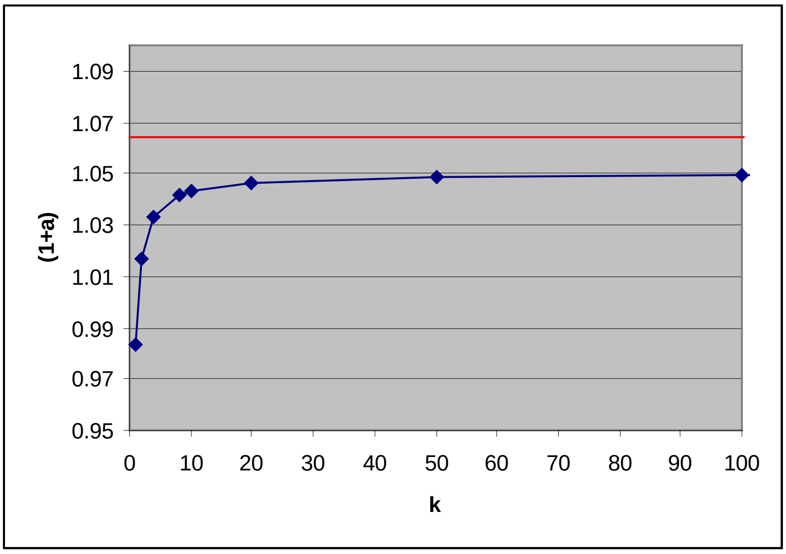
[2] Martin Gasche (2012), Alte und neue Wege zur Berechnung der Rentenabschläge, Discussion Paper, MEA, Munich.

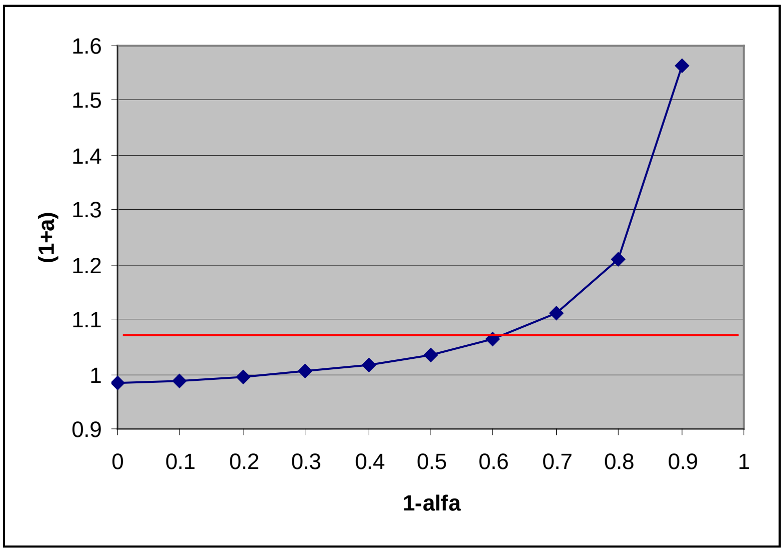
such that

(12) (1+a) = (exp[log{p+Dp}+alfa/(1-alfa)*{log(D+1)-log(D)}]-(1-b)*y) / Dp

(1+a) resulting from equation (12) can be higher or lower than (1+a) derived from the PDV-based calculation in equation (6) depending on the relative weight alfa for consumption versus leisure, see figure below. A very high weight on consumption yields a lower utility-based adjustment factor, while a very high weight on leisure yields a higher utility-based adjustment factor, relative to the PDV-based adjustment factor. As opposed to the result derived from using the SW utility function, this is in accordance to economic intuition.

The figures below show the implied adjustment factors (1+a) for various values of k (SW utility function, upper graph)) and alfa (log utility function, lower graph) with D=20, b=0.2, and r=0.6. The PDV-based value for (1+a) derived from equation (4) is 1.067 and marked by the red line in both figures.





Finally, the SW specification of utility has implications for the econometric estimation of retirement behavior. For the commonly used parameter values, the SW utility function is almost horizontal in the (c,l)-plane, see the figure above. This is much different in the log utility function. If the true demand for leisure is of a type best described by the log utility function but the econometric model assumes a SW utility function, then the estimated parameters of the leisure-consumption trade off in the retirement model are biased as the implied demand function for leisure is misspecified.

The option value of postponing retirement is the difference between the utility when retiring at the age that maximizes utility minus the utility when retiring now. A utility function which depends only weakly on leisure (i.e. retirement age) such as the SW utility function makes this difference flat relative to a more leisure-sensitive utility function. This may explain the poor results observed in many countries which have participated in the Gruber-Wise exercise where the option value was based on the SW utility function.